AS Mathematics Unit 1: Pure Mathematics A

General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through (✓ indicates correct working following an error and ✗ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

5. Marking codes

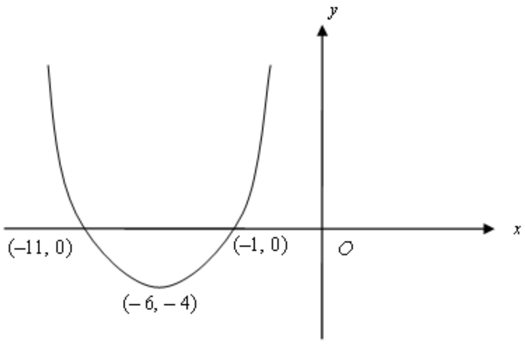
- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

AS Mathematics Unit 1: Pure Mathematics A

Solutions and Mark Scheme

Question Number	Solution	Mark	AO	Notes
1. (a)	$A(1, -3)$ A correct method for finding the radius, e.g., trying to rewrite the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ Radius = 5	B1 M1 A1	AO1 AO1 AO1	
(b)	Gradient $AP = \dfrac{\text{increase in } y}{\text{increase in } x}$ Gradient $AP = \dfrac{(-7)-(-3)}{4-1} = -\dfrac{4}{3}$ Use of $m_{\tan} \times m_{\text{rad}} = -1$ Equation of tangent is: $y-(-7) = \dfrac{3}{4}(x-4)$	M1 A1 M1 A1 **[7]**	AO1 AO1 AO1 AO1	(f.t. candidate's coordinates for *A*) (f.t. candidate's gradient for *AP*)
2.	$7\sin^2\theta + 1 = 3(1-\sin^2\theta) - \sin^2\theta$ An attempt to collect terms, form and solve a quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant $10\sin^2\theta + \sin\theta - 2 = 0$ $\Rightarrow (2\sin\theta + 1)(5\sin\theta - 2) = 0$ $\Rightarrow \sin\theta = -\dfrac{1}{2}, \sin\theta = \dfrac{2}{5}$ $\theta = 210°, 330°$ $\theta = 23{\cdot}57(8178\ldots)°, 156{\cdot}42(182\ldots)°$ Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range. $\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks $\sin\theta = +, +$, f.t. for 1 mark	M1 m1 A1 B1 B1 B1 **[6]**	AO1 AO1 AO1 AO1 AO1 AO1	(correct use of $\cos^2\theta = 1-\sin^2\theta$) (c.a.o.)

Question Number	Solution	Mark	AO	Notes
3.	$y + k = (x + h)^3$	M1	AO2	
	$y + k = x^3 + 3x^2h + 3xh^2 + h^3$	A1	AO2	
	Subtracting y from above to find k	M1	AO2	
	$k = 3x^2h + 3xh^2 + h^3$	A1	AO2	
	Dividing by h and letting $h \to 0$	M1	AO2	
	$\dfrac{dy}{dx} = \underset{h \to 0}{\text{limit}} \dfrac{k}{h} = 3x^2$	A1	AO2	(c.a.o.)
		[6]		
4.	Correct use of the Factor Theorem to find at least one factor *of* $f(x)$	M1	AO3	
	At least two factors of $f(x)$	A1	AO3	(accept $(x – 2{\cdot}5)$ as a factor)
	$f(x) = (x + 3)(x – 4)(2x – 5)$	A1	AO3	(c.a.o.)
	Use of the fact that $f(x)$ intersects the y-axis when $x = 0$	M1	AO3	
	$f(x)$ intersects the y-axis at (0, 60)	A1	AO3	(f.t. candidate's expression for $f(x)$)
		[5]		
5. (a)	A correct method for finding the coordinates of the mid-point of *AB*	M1	AO1	
	D has coordinates *(– 1, 5)*	A1	AO1	
	Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$	M1	AO1	
	Gradient of $AB = -\dfrac{6}{2}$	A1	AO1	(or equivalent)
	Gradient of $CD = \dfrac{\text{increase in } y}{\text{increase in } x}$	(M1)	(AO1)	(to be awarded only if the previous M1 is not awarded)
	Gradient of $CD = \dfrac{7}{21}$	A1	AO1	(or equivalent)
	$-\dfrac{6}{2} \times \dfrac{7}{21} = -1 \Rightarrow$ *AB* is perpendicular to *CD*	B1	AO2	
(b)	A correct method for finding the length of *AD* or *CD*	M1	AO1	
	$AD = \sqrt{10}$	A1	AO1	
	$CD = \sqrt{490}$	A1	AO1	
	$\tan C\hat{A}B = \dfrac{CD}{AD}$	M1	AO1	
	$\tan C\hat{A}B = 7$	A1	AO1	
(c)	Isosceles	B1	AO2	
		[12]		

Question Number	Solution	Mark	AO	Notes
6. (a)	For statement A Choice of $c \neq -{}^{1}/_{2}$ and $d = -c - 1$ Correct verification that given equation is satisfied	M1 A1	AO2 AO2	
(b)	For statement B Use of the fact that any real number has an unique real cube root $(2c + 1)^3 = (2d + 1)^3 \Rightarrow 2c + 1 = 2d + 1$ $2c + 1 = 2d + 1 \Rightarrow c = d$	M1 A1 A1 **[5]**	AO2 AO2 AO2	
7. (a)				
	Concave up curve and y-coordinate of minimum = – 4 x-coordinate of minimum = – 6 Both points of intersection with x -axis	B1 B1 B1	AO1 AO1 AO1	
(b)	$y = -\frac{1}{2} f(x)$ **If B2 not awarded** $y = rf(x)$ with r negative	B2 (B1) **[5]**	AO2 AO2 (AO2)	

Question Number	Solution	Mark	AO	Notes
8. (a)	A kite	B1	AO2	
(b)	A correct method for finding $TR(TS)$	M1	AO3	
	$TR(TS) = \sqrt{96}$	A1	AO3	
	Area $OTR(OTS) = \frac{1}{2} \times \sqrt{96} \times 5$	M1	AO3	(f.t. candidate's derived value for $TR(TS)$)
	Area $OTRS = 2 \times$ Area $OTR(OTS)$	m1	AO3	
	Area $OTRS = 20\sqrt{6}$	A1 **[6]**	AO3	(c.a.o.)
9.	An expression for $b^2 - 4ac$ for the quadratic equation $4x^2 - 12x + m = 0$, with at least two of a, b or c correct	M1	AO1	
	$b^2 - 4ac = 12^2 - 4 \times 4 \times m$	A1	AO1	
	$b^2 - 4ac > 0$	m1	AO1	
	$(0<)\ m < 9$	A1	AO1	
	An expression for $b^2 - 4ac$ for the quadratic equation $3x^2 + mx + 7 = 0$, with at least two of a, b or c correct	(M1)		(to be awarded only if the corresponding M1 is not awarded above)
	$b^2 - 4ac = m^2 - 84$	A1	AO2	
	$m^2 < 81 \Rightarrow b^2 - 4ac < -3$	A1	AO2	
	$b^2 - 4ac < 0 \Rightarrow$ no real roots	A1 **[7]**	AO2	
10. (a)	$(\sqrt{3} - \sqrt{2})^5 = (\sqrt{3})^5 + 5(\sqrt{3})^4(-\sqrt{2}) + 10(\sqrt{3})^3(-\sqrt{2})^2 + 10(\sqrt{3})^2(-\sqrt{2})^3 + 5(\sqrt{3})(-\sqrt{2})^4 + (-\sqrt{2})^5$	B2	AO1 AO1	(five or six terms correct)
	(**If B2 not awarded**, award B1 for three or four correct terms)			
	$(\sqrt{3} - \sqrt{2})^5 = 9\sqrt{3} - 45\sqrt{2} + 60\sqrt{3} - 60\sqrt{2} + 20\sqrt{3} - 4\sqrt{2}$	B2	AO1 AO1	(six terms correct)
	(**If B2 not awarded**, award B1 for three, four or five correct terms)			
	$(\sqrt{3} - \sqrt{2})^5 = 89\sqrt{3} - 109\sqrt{2}$	B1	AO1	(f.t. one error)
(b)	Since $(\sqrt{3} - \sqrt{2})^5 \approx 0$, we may assume that $89\sqrt{3} \approx 109\sqrt{2}$	M1	AO3	(f.t candidate's answer to part (a) provided one coefficient is negative)
	Either: $89\sqrt{3} \times \sqrt{3} \approx 109\sqrt{2} \times \sqrt{3}$	m1	AO3	(f.t candidate's answer to part (*a*) provided one coefficient is negative)
	$\sqrt{6} \approx \frac{267}{109}$	A1	AO3	(c.a.o.)
	Or $89\sqrt{3} \times \sqrt{2} \approx 109\sqrt{2} \times \sqrt{2}$	(m1)	(AO3)	(f.t candidate's answer to part (*a*) provided one coefficient is negative)
	$\sqrt{6} \approx \frac{218}{89}$	(A1) **[8]**	(AO3)	(c.a.o.)

Question Number	Solution	Mark	AO	Notes
11.	$a > 0$ $b > a + 2$ $b < 6 + 4a - a^2$	B1 B1 B1 **[3]**	AO1 AO1 AO1	
12.	Let $p = \log_a 19$, $q = \log_7 a$ Then $19 = a^p$, $a = 7^q$ $19 = a^p = \left(7^q\right)^p = 7^{qp}$ $qp = \log_7 19$ $\log_7 a \times \log_a 19 = \log_7 19$	 B1 B1 B1 **[3]**	 AO2 AO2 AO2	(the relationship between log and power) (the laws of indices) (the relationship between log and power) (convincing)
13. (a)	Choice of variable (x) for $AB \Rightarrow AC = x + 2$ $(x+2)^2 = x^2 + 12^2 - 2 \times x \times 12 \times \frac{2}{3}$ $x^2 + 4x + 4 = x^2 + 144 - 16x$ $20x = 140 \Rightarrow x = 7$ $AB = 7$, $AC = 9$	B1 M1 A1 A1	AO3 AO3 AO3 AO3	 (Amend proof for candidates who choose $AC = x$)
(b)	$\sin A\hat{B}C = \frac{\sqrt{5}}{3}$ $\frac{\sin B\hat{A}C}{12} = \frac{\sin A\hat{B}C}{9}$ $\sin B\hat{A}C = \frac{4\sqrt{5}}{9}$	B1 M1 A1 **[7]**	AO1 AO1 AO1	 f.t. candidate's derived values for AC and $\sin A\hat{B}C$) (c.a.o.)
14. (a)	Height of box $= \frac{9000}{2x^2}$ $S = 2 \times (2x \times x + \frac{9000}{2x^2} \times x + \frac{9000}{2x^2} \times 2x$ $S = 4x^2 + \frac{27000}{x}$	B1 M1 A1	AO3 AO3 AO3	(o.e.) (f.t. candidate's derived expression for height of box in terms of x) (convincing)
(b)	$\frac{dS}{dx} = 8x - \frac{27000}{x^2}$ Putting derived $\frac{dS}{dx} = 0$ $x = 15$ Stationary value of S at $x = 15$ is 2700 A correct method for finding nature of the stationary point yielding a minimum value	B1 M1 A1 A1 B1 **[8]**	AO1 AO1 AO1 AO1 AO1	 (f.t. candidate's $\frac{dS}{dx}$) (c.a.o)

Question Number		Solution	Mark	AO	Notes
15.	(a)	A represents the initial population of the island.	B1	AO3	
	(b)	$100 = Ae^{2k}$ $160 = Ae^{12k}$	B1	AO1	(both values)
		Dividing to eliminate A	M1	AO1	
		$1{\cdot}6 = e^{10k}$	A1	AO1	
		$k = \frac{1}{10}\ln 1.6 = 0.047$	A1	AO1	(convincing)
	(c)	A = 91(·0283)	B1	AO1	(o.e.)
		When t = 20, N = 91(·0283) $\times e^{0{\cdot}94}$	M1	AO1	(f.t. candidate's derived value for A)
		N = 233	A1 **[8]**	AO3	(c.a.o.)
16.		$f'(x) = 3x^2 - 10x - 8$	M1	AO1	(At least one non-zero term correct)
		Critical values $x = -\frac{2}{3}$, $x = 4$	A1	AO1	(c.a.o)
		For an increasing function, $f'(x) > 0$	m1	AO1	
		For an increasing function $x < -\frac{2}{3}$ or $x > 4$	A2	AO2 AO2	(f.t. candidate's derived two critical values for x)
		Deduct 1 mark for each of the following errors the use of non-strict inequalities the use of the word 'and' instead of the word 'or'	**[5]**		

Question Number			Solution	Mark	AO	Notes
17.	(a)		$\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - 2x$	M1	AO1	(At least one non-zero term correct)
			An attempt to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$	m1	AO1	
			At $x = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -1$	A1	AO1	(c.a.o.)
			Equation of tangent at *B* is $y - 2 = -1(x - 2)$	A1	AO1	(f.t. candidate's value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$)
	(b)		x-coordinate of *A* = 3	B1	AO1	(derived)
			x-coordinate of *C* = 4	B1	AO1	(derived)
			If *D* is the foot of the perpendicular from *B* to the x-axis, area of triangle *BDC* = 2	B1	AO1	(f.t. candidate's derived x-coordinate of *C*)
			Area under curve = $\int_2^3 (3x - x^2)\mathrm{d}x$	M1	AO3	(use of integration) (f.t. candidate's derived x-coordinate of *A*)
			$\frac{3x^2}{2} - \frac{x^3}{3}$	A1	AO3	(correct integration)
			Area under curve = (27/2 – 9) – (6 – 8/3)	m1	AO3	(an attempt to substitute limits, f.t. candidate's derived x-coordinate of *A*)
			Shaded area = Area of triangle *BDC* – Area under curve	m1	AO3	(f.t. candidate's derived x-coordinates of *A* and *C*)
			Shaded area = 5/6	A1 **[12]**	AO3	(c.a.o.)
18.	(a)	(i)	$4\mathbf{u} - 3\mathbf{v} = 20\mathbf{i} - 27\mathbf{j}$	B1 B1	AO1 AO1	
		(ii)	A correct method for finding the length of *UV*	M1	AO1	
			Length of *UV* = 10	A1	AO1	
	(b)	(i)	Position vector of $C = \frac{1}{10}\mathbf{a} + \frac{9}{10}\mathbf{b}$ or $C = \frac{9}{10}\mathbf{a} + \frac{1}{10}\mathbf{b}$	M1	AO3	
			Position vector $C = \frac{1}{10}\mathbf{a} + \frac{9}{10}\mathbf{b}$	A1	AO3	
		(ii)	The position vector of any point on the road will be of the form $\lambda\mathbf{a} + (1 - \lambda)\mathbf{b}$ for some value of λ	B1 **[7]**	AO2	